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A note on the paper "Analysis of perfectly bonded wedges and bonded wedges with an interfacial crack under antiplane shear loading" ☆

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Analytical solution of the antiplane deformation problem of the two bonded wedges with equal apex angles containing an interfacial crack, in Shahani and Adibnazari (2000), leads to (Eq. (52) in that reference)

$$f(r) = \frac{P}{\mu_e \alpha}\sqrt{h^\gamma(a^\gamma + h^\gamma)(b^\gamma + h^\gamma)}\left[\frac{1}{r^\gamma + h^\gamma} - \frac{1}{b^\gamma + h^\gamma}\frac{\prod(k,n,\frac{\pi}{2})}{K(k,\frac{\pi}{2})}\right] r^{(\gamma/2)-1}\left[(r^\gamma - a^\gamma)(b^\gamma - r^\gamma)\right]^{-1/2}; \quad a \leqslant r \leqslant b \tag{1}$$

where $\mu_e = \mu_1\mu_2/(\mu_1 + \mu_2)$ and

$$k^2 = \frac{b^\gamma - a^\gamma}{b^\gamma}, \qquad n = -\frac{b^\gamma - a^\gamma}{b^\gamma + h^\gamma} \tag{2}$$

and $K(k, \pi/2)$ and $\prod(k, n, \pi/2)$ are the complete elliptic integrals of the first and third kinds, respectively (Spiegel, 1968) and are given by the relations

$$K\left(k, \frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\theta}{\sqrt{1 - k^2 \sin^2\theta}}; \quad 0 \leqslant k \leqslant 1 \tag{3}$$

$$\prod\left(k, n, \frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\theta}{(1 + n\sin^2\theta)\sqrt{1 - k^2\sin^2\theta}}; \quad 0 \leqslant k \leqslant 1 \tag{4}$$

Here, it is desired to examine the behaviour of $f(r)$ as the crack tip $r = a$ approaches the wedge apex ($a \to 0$). It is seen that the stress singularity approaches unity as $a \to 0$ in Eq. (1), but it will be shown that it never exceeds unity. For this purpose, let us examine the behaviour of K and $\prod$ as $a \to 0$ (i.e., $k^2 \to 1$ and $n \to n_0 = -(b^\gamma/(b^\gamma + h^\gamma))$).

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Adding and subtracting a term $n\sin^2\theta$ to and from the numerator of Eq. (4), gives:

$$\prod\left(k,n,\frac{\pi}{2}\right)=\int_0^{\frac{\pi}{2}}\frac{1+n\sin^2\theta-n\sin^2\theta}{(1+n\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}\mathrm{d}\theta=K\left(k,\frac{\pi}{2}\right)-\int_0^{\frac{\pi}{2}}\frac{n\sin^2\theta\,\mathrm{d}\theta}{(1+n\sin^2\theta)\sqrt{1-k^2\sin^2\theta}} \tag{5}$$

In the limit:

$$\begin{aligned}\lim_{a\to 0}\prod\left(k,n,\frac{\pi}{2}\right)&=\prod\left(1,n_0,\frac{\pi}{2}\right)=K\left(1,\frac{\pi}{2}\right)-\int_0^{\frac{\pi}{2}}\frac{n_0\sin^2\theta\,\mathrm{d}\theta}{(1+n_0\sin^2\theta)\cos\theta}\\&=K\left(1,\frac{\pi}{2}\right)-\int_0^{\frac{\pi}{2}}\frac{n_0(1-\cos^2\theta)\mathrm{d}\theta}{(1+n_0\sin^2\theta)\cos\theta}\\&=K\left(1,\frac{\pi}{2}\right)-n_0\prod\left(1,n_0,\frac{\pi}{2}\right)+n_0\int_0^{\frac{\pi}{2}}\frac{\cos\theta\,\mathrm{d}\theta}{1+n_0\sin^2\theta}\end{aligned} \tag{6}$$

or after simplification:

$$(1+n_0)\prod\left(1,n_0,\frac{\pi}{2}\right)=K\left(1,\frac{\pi}{2}\right)+n_0I \tag{7}$$

where

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos\theta\,\mathrm{d}\theta}{1+n_0\sin^2\theta} \tag{8}$$

The above integral may be computed using the change of variable $u=\sin\theta$, as

$$I=\int_0^1\frac{\mathrm{d}u}{1+n_0u^2}=\frac{1}{\sqrt{n_0}}\tan^{-1}(\sqrt{n_0}u)\Big|_0^1=\frac{1}{\sqrt{n_0}}\tan^{-1}(\sqrt{n_0}) \tag{9}$$

where

$$\sqrt{n_0}=\mathrm{i}\beta\qquad\beta=\sqrt{\frac{b^\gamma}{b^\gamma+h^\gamma}},\quad\beta<1 \tag{10}$$

It may be shown that the expression in Eq. (9) and thus, the second term of Eq. (7) is a finite real number:

$$\sqrt{n_0}\tan^{-1}(\sqrt{n_0})=\mathrm{i}\beta\tan^{-1}(\mathrm{i}\beta)=\mathrm{i}\beta\mathrm{i}\tanh^{-1}\beta=-\beta\tanh^{-1}\beta \tag{11}$$

Substituting from Eq. (11) into Eq. (7) after dividing both sides of it by $K(1,\pi/2)$, gives

$$(1+n_0)\frac{\prod(1,n_0,\frac{\pi}{2})}{K(1,\frac{\pi}{2})}=1-\frac{\beta\tanh^{-1}\beta}{K(1,\frac{\pi}{2})} \tag{12}$$

Noting that $K(1,\pi/2)\to\infty$ and remembering that $\beta\tanh^{-1}(\beta)$ is a finite real number, we have

$$\frac{\prod(1,n_0,\frac{\pi}{2})}{K(1,\frac{\pi}{2})}=\lim_{a\to 0}\frac{\prod(k,n,\frac{\pi}{2})}{K(k,\frac{\pi}{2})}=\frac{1}{1+n_0} \tag{13}$$

Substituting this together with $a=0$ into Eq. (1), and simplify result in

$$f(r)=-\frac{P\sqrt{b^\gamma+h^\gamma}}{\mu_e\theta_1}\frac{r^{\gamma-1}(b^\gamma-r^\gamma)^{-1/2}}{r^\gamma+h^\gamma}\quad\text{as }\ a\to 0 \tag{14}$$

Since $\theta_1\leqslant\pi$, we have: $\gamma=\pi/\theta_1\geqslant 1$ and thus, it is observed that the singularity at the apex vanishes as the crack tip $r=a$ coincides with the wedge apex. On the other hand, the stress distribution exhibits the familiar square root singularity at the crack tip $r=b$.

References

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